Module 1

COMPOSITION AND RESOLUTION OF FORCES {Ref: EM- R.S.Khurmi

Composition / Compounding:

The process of finding out the resultant of a number of forces is called composition or compounding of forces.

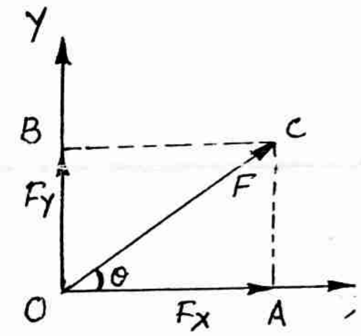
Resultant:

It is a single force which will have the same effect as the system of forces.

Resolution:

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. Resolution is the reverse process of finding the resultant force.

Resolution of a Force:



Let a given force- 'F' makes an angle -θ with x-axis. To resolve 'F' means, to find the components of 'F' along x-axis & y-axis.

Consider the rectangle OACB,

$$\sin\theta = \frac{AC}{OC} = \frac{OB}{OC} = \frac{F_Y}{F}$$

$$\therefore F_Y = F\sin\theta$$

ie, Component of F along y-axis, $F_Y = F\sin\theta$

$$\cos\theta = \frac{OA}{OC} = \frac{F_X}{F}$$

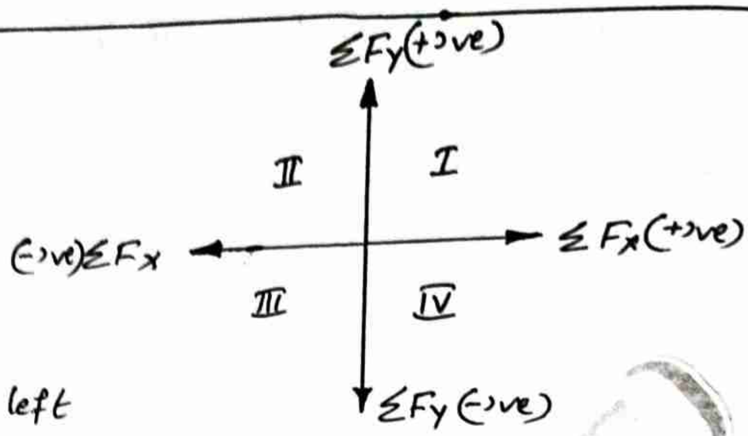
ie, Component of F along x-axis, $F_X = F\cos\theta$

The method of resolution of force is also known as <u>Method of Projection</u>.

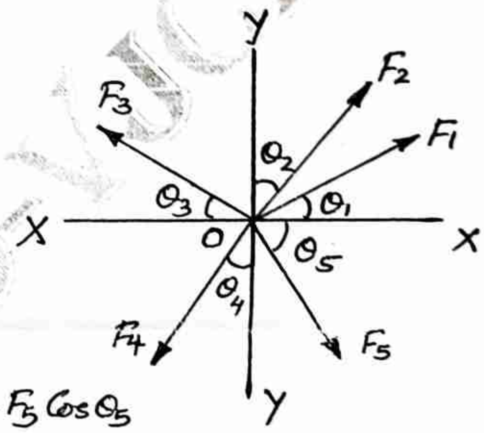
Method of Resolution for the Resultant of a system of Coplanar Concurrent Forces:

Sign Convention:

- Forces towards right & forces in upward dirn is taken as +ve.
- Forces towards left & forces in downward dirn is taken as -ve.



Procedure:



* Resolve all the forces horizontally and find the algebraic sum of all the horizontal components.

ie, ΣF_x or $\Sigma H = F_1 \cos\theta_1 + F_2 \sin\theta_2 - F_3 \cos\theta_3 - F_4 \sin\theta_4 + F_5 \cos\theta_5$

* Resolve all the forces vertically and find the algebraic sum of all the vertical components.

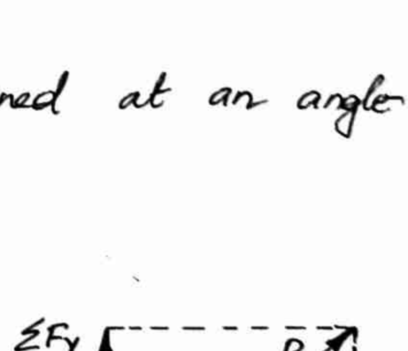
ie, ΣF_y or $\Sigma V = F_1 \sin\theta_1 + F_2 \cos\theta_2 + F_3 \sin\theta_3 - F_4 \cos\theta_4 - F_5 \sin\theta_5$

* The resultant -'R' of the given forces will be given by the equation, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

* The resultant force will be inclined at an angle with the horizontal such that,

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

$$\therefore \theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$



Note:-

- When ΣF_x and ΣF_y are +'ve, 'R' is in I quadrant.
- When ΣF_x is -'ve and ΣF_y is +'ve, 'R' is in II quadrant
- When ΣF_x and ΣF_y are -'ve, 'R' is in III quadrant.
- When ΣF_x is +'ve and ΣF_y is -'ve, 'R' is in IV quadrant

PRINCIPLE OF RESOLUTION:

It states that 'The algebraic sum of the resolved parts of a no: of forces, in a given direction is equal to the resolved part of their resultant in the same direction'.

Let 'R' - Resultant of the given forces

R_x - Component of the resultant in x-direction

R_y - Component of the resultant in y-direction

ΣF_x - Algebraic sum of the horizontal components of the given forces.

ΣF_y - Algebraic sum of the vertical components of the given forces.

According to the principle of resolution,

$$\Sigma F_x = R_x$$

&

$$\Sigma F_y = R_y$$

• <u>Equilibrium of Rigid Body:</u> {Ref: EM- Bansal, Benjamin

A rigid body is said to be in equilibrium if resultant of the forces and resultant moment are zero. The necessary and sufficient conditions for the equilibrium of a rigid body are;

$$\left.\begin{array}{l}\Sigma F_x = 0 \\ \Sigma F_y = 0\end{array}\right\} \text{ ie, } R = 0$$

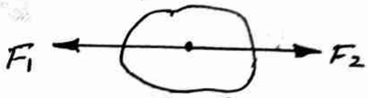
$$\Sigma M = 0$$

These are the conditions of equilibrium.

<u>Conditions of Equilibrium in two dimensions:</u>

i) <u>Two Force System:-</u>

When a body is subjected to two forces, then the body will be in equilibrium if the two forces are collinear equal and opposite.

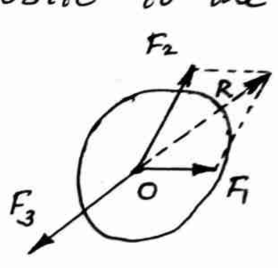


$F_1 = F_2$

ii) <u>Three Force System:-</u>

When a body is subjected to three concurrent force then the body will be in equilibrium, if the resultant of the two forces is equal and opposite to the third for

The three forces (concurrent) - F_1, F_2 & F_3 are acting on a body at 'O' and the body is in equilibrium. The resultant of F_1 and F_2 is given by 'R'. If the force - F_3 is collinear, equal and opposite to the resultant - R, then the body will be in equilibrium. The force - F_3 which is equal and opposite to the resultant 'R' is known as equilibrant.



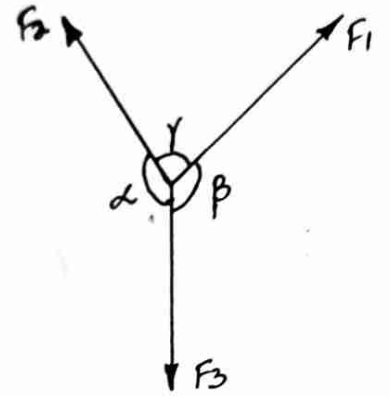
<u>Lami's theorem:</u> (Three Co-planar Concurrent Forces) {Ref. EM- Ramamrutham, Bhavikatti

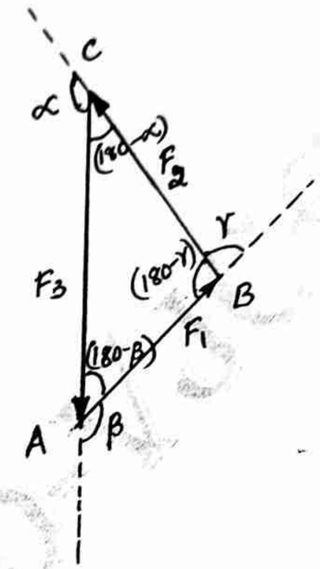
If a body is in equilibrium under the action of only three forces, Lami's theorem can be used.

Lami's theorem states that, 'If three co-planar concurrent forces are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.'

ie, For the force system shown in the figure,

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$





<u>Proof:-</u>

Consider three co-planar concurrent forces F_1, F_2 and F_3 as shown. These three forces will be in equilibrium if their resultant is zero. For this, the force polygon must be a closed one. ie, the force polygon must be a triangle. Since AB, BC & CA are parallel to F_1, F_2 and F_3, the external angles at A, B and C are β, γ and α.

In the force ΔABC, applying Sine rule,

$$\frac{F_1}{\sin(180-\alpha)} = \frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\gamma)}$$

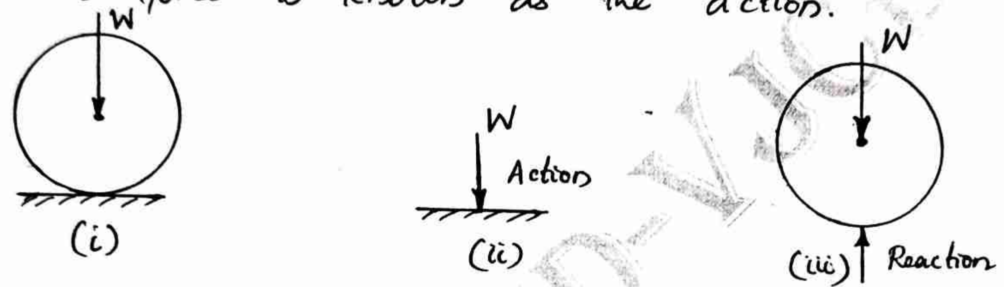
ie,

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Action and Reaction:

According to Newton's 3rd law, to every action, there an equal and opposite reaction. Hence reaction is always eq and opposite to the action.

Fig(i) shows a ball placed on a smooth frictionless horiz surface such that it is free to move along the plane but cannot move vertically downward. Here, the ball will exert a force vertically downwards at the support as shown in fig and this force is known as the action.



The support will exert an equal force vertically upwards on the ball at the point of contact as shown in fig(iii). The force exerted by the support on the ball is known as reactio Reaction will always be normal to the surface.

Free body diagram:

{ Ref: EM - Tayal, Timoshenko, Bhavikatti

A diagram of the body in which the body under considera is freed from all the contact surfaces and showing all the forces actin on it (including reactions at contact surfaces) ~~are drawn~~ is called a free body diagram (FBD).

To draw the free body diagram of a body, we remove all the supports (like wall, hinge, floor or any other body) and replace them by the reactions which these supports exert on the body

The general procedure for constructing a free body diagram is given below;

- A sketch of the body is drawn, by removing the supporting surfaces.

- Indicate all the applied forces and self-weight of the bo
- Indicate all the reactions.
- All the important dimensions, angles, reference axes are al shown in the sketch.

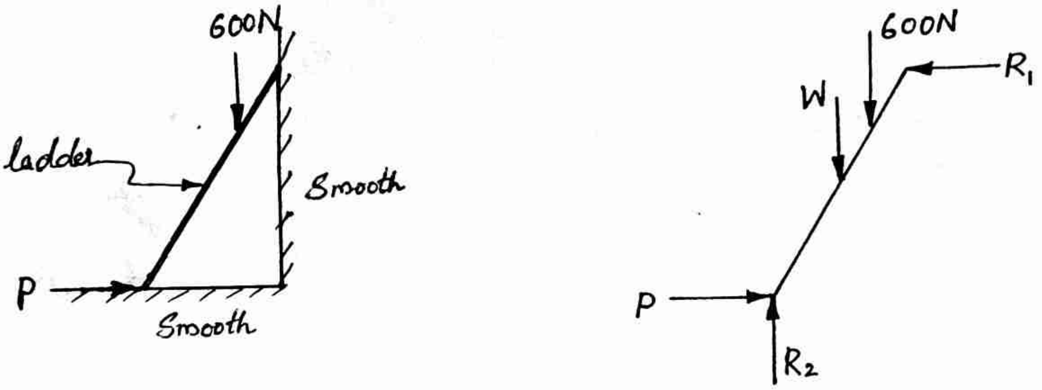
FBD for few typical cases:

(Reacting bodies) (Free body diagram)

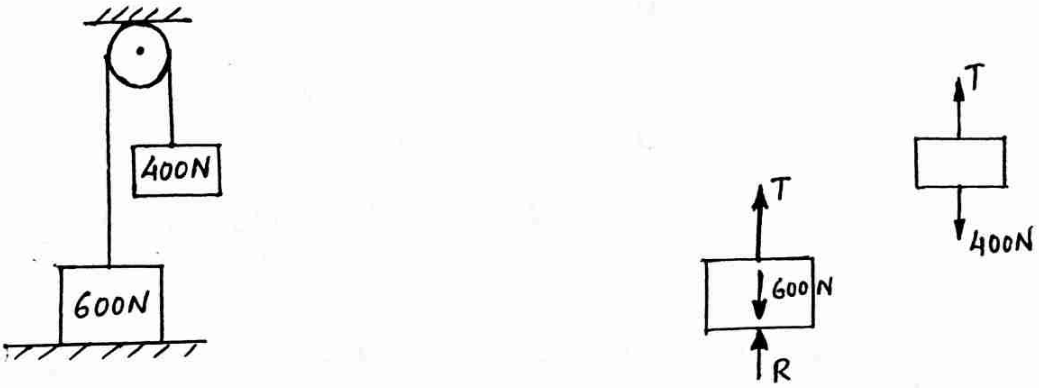
i)

ii)

iii)



iv)

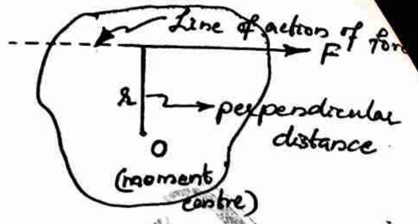


MOMENT OF A FORCE {Ref: EM- R.K. Bansal

The moment of a force about a point is the product force and the perpendicular distance between the point an the line of action of the force.

Let F = a force acting on a body.

r = perpendicular distance from the point-'O' on the line of action of force-'F'. (moment arm)



Moment of the force F about-O, $\boxed{M = F \times r}$

Unit - Nm

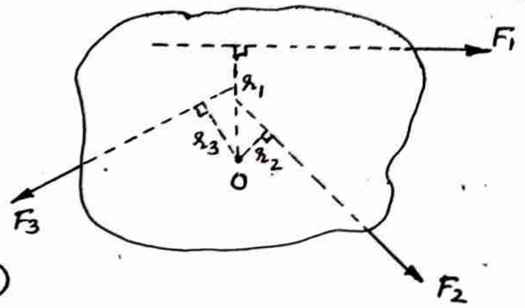
The tendency of the moment of a force about a point is to turn the body on which it is acting about the point. If the tendency of a moment is to rotate the body in the clockwise direction, then the moment is called clockwise moment. If the tendency of a moment is to rotate the body in anti-clockwise direction, then the moment is known as anti-clockwise moment.

If clockwise moment is taken +ve then anti-clockwise moment will be -ve.

Resultant moment of a number of Forces:

Consider a body on which three forces - F_1, F_2, F_3 are acting as shown in fig.

Let r_1, r_2, r_3 be ⊥r distances of the line of action of forces F_1, F_2 & F_3 respectively from the point-'O'.



Moment of F_1 about $O = F_1 \times r_1$ (+ve)

Moment of F_2 about $O = F_2 \times r_2$ (+ve)

Moment of F_3 about $O = F_3 \times r_3$ (-ve)

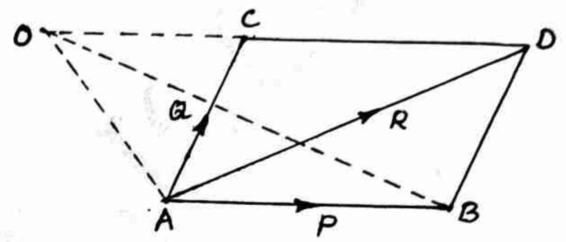
The resultant moment will be the algebraic sum of all the moments.

∴ Resultant moment of F_1, F_2 & F_3 about 'O' = $\underline{F_1 r_1 + F_2 r_2 - F_3 r_3}$.

PRINCIPLE OF MOMENTS OR VARIGNON'S PRINCIPLE {Ref: EM-D.S.Kum

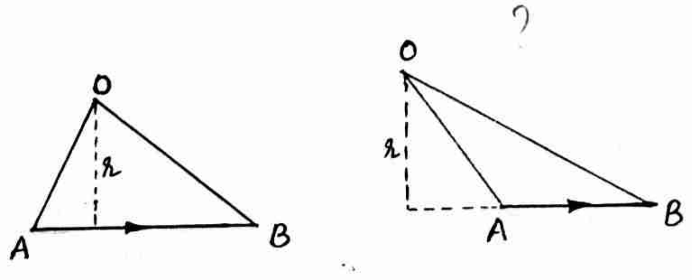
Principle of moments states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.

According to Varignon's principle, the moment of a force about any point is equal to the algebraic sum of the moment of its components about that point.



Consider 2 concurrent forces P and Q represented in magnitude and direction by AB & AC resply. Let 'O' be the point about which moment is to be taken. Through-O, draw a line parallel to the direction of force-'P' and let this line meet the line of action of force-'Q' at point-C. With AB & AC as adjacent sides, complete the parallelogram ABDC. The diagonal-AD of the parallelogram represents in magnitude and direction, the resultant of forces P & Q. Join 'O' with points A & B.

[Moment of a force about a point is equal to twice the area of the triangle so formed whose base is the line that represents the force and whose vertex is the point about which the moment is required to be found out.



Graphical representation of moment.]

We have,

Moment of force-P about O = 2 x area of ΔAOB

Moment of force-Q about O = 2x area of ΔAOC

Moment of force-R about O = 2x area of ΔAOD

From the geometry of the fig.,

$\Delta AOD = \Delta AOC + \Delta ACD$

$\quad = \Delta AOC + \Delta ABD$

Since ΔAOB and ΔABD are on the same base-AB and between the same lines, they are equal in area.

Then $\Delta AOD = \Delta AOC + \Delta AOB$.

The moment of force-R about-'O' = 2 × area of ΔAOD

= 2 × area ($\Delta AOC + \Delta AOB$)

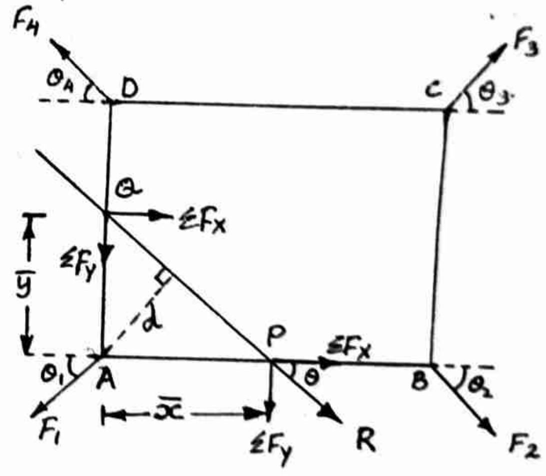
= 2× area of ΔAOC + 2× area Δ AOB

= Moment of 'Q' about O + Moment of 'P' about O.

∴ Moment of forces P & Q about point-'O'
= Moment of resultant-R about 'O'.

- **Locating the resultant of non-concurrent forces:**

Consider a body on which forces F_1, F_2, F_3 & F_4 are acting as shown.



The method of finding the resultant and its position is given as follows:

i) Find ΣF_x

$$\Sigma F_x = -F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 - F_4 \cos\theta_4$$

ii) Find ΣF_y

$$\Sigma F_y = -F_1 \sin\theta_1 - F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4$$

iii) Calculate the magnitude of resultant-R and its inclination with the horizontal.

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

iv) Let 'A' be the point about which the resultant is to be located. Let 'd' be the perpendicular distance of line of action of resultant from 'A'.

Taking moments of all forces about 'A' and applying Varignon's theorem, we get,

$R \times d = \Sigma M_A$; where A - moment centre

ie, $d = \dfrac{\Sigma M_A}{R}$ d - moment arm

ΣM_A - Sum of moment of all forces about A.

v) Check the signs of ΣF_x, ΣF_y and ΣM_A and locate 'R'.

* If we have obtained ΣF_x as +ve and ΣF_y as -ve and ΣM_A as clockwise, 'R' will be located as shown in fig

Let the line of action of R cut the x and y-axis through A at P and Q respectively.

Let $\bar{x} = AP = x$ intercept

$\bar{y} = AQ = y$ intercept

Considering the forces acting at P and taking moments about A, we get,

$$\Sigma F_y \times \bar{x} = \Sigma M_A$$

$$\therefore \bar{x} = \frac{\Sigma M_A}{\Sigma F_y}$$

Similarly, considering the forces acting at Q and taking moments about A,

$$\Sigma F_x \times \bar{y} = \Sigma M_A$$

$$\therefore \bar{y} = \frac{\Sigma M_A}{\Sigma F_x}$$

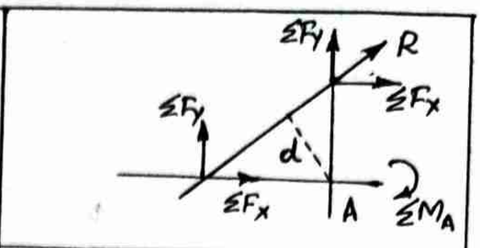
Note:

For a parallel force system, resultant force will be parallel to all forces and at a distance of

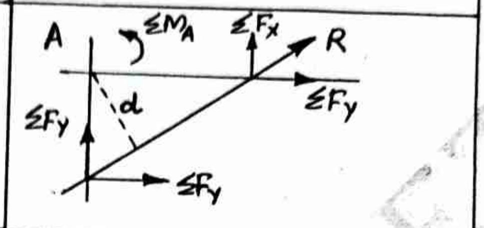
x or $d = \dfrac{\Sigma M_A}{R}$ from the moment centre.

POSITION OF RESULTANT:

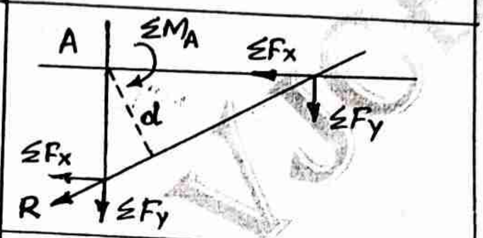
Case 1: ΣF_x +ve, ΣF_y +ve, ΣM_A ↻



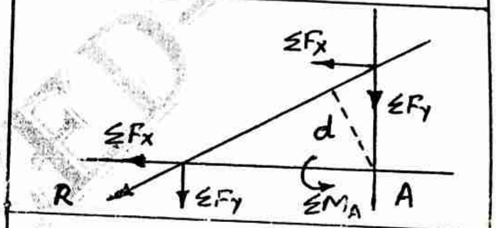
Case 2: ΣF_x +ve, ΣF_y +ve, ΣM_A ↺



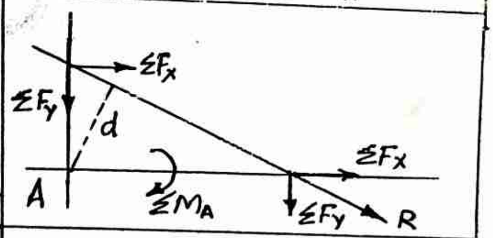
Case 3: ΣF_x −ve, ΣF_y −ve, ΣM_A ↻



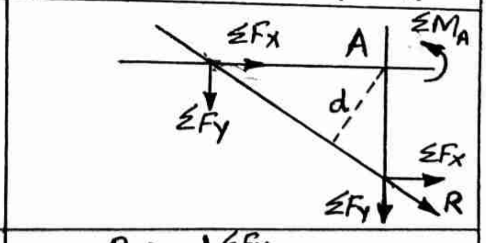
Case 4: ΣF_x −ve, ΣF_y −ve, ΣM_A ↺



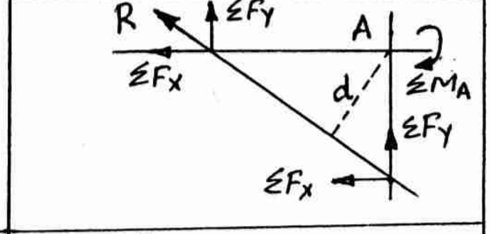
Case 5: ΣF_x +ve, ΣF_y −ve, ΣM_A ↻



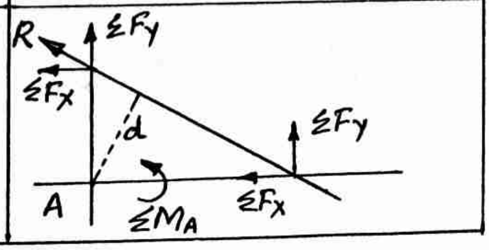
Case 6: ΣF_x +ve, ΣF_y −ve, ΣM_A ↺



Case 7: ΣF_x −ve, ΣF_y +ve, ΣM_A ↻

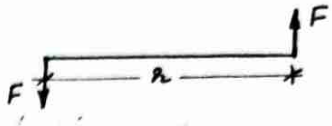


Case 8: ΣF_x −ve, ΣF_y +ve, ΣM_A ↺



Couple { Ref: EM- R.S. Khurmi, D.S. Kumar

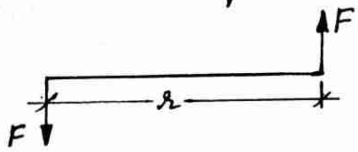
A pair of two equal and unlike parallel forces (ie, forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.



The rotational effect of a couple is measured by its moment which is defined as the product of either of the forces and the perpendicular distance between the forces. The perpendicular distance separating the two forces is called the arm of the couple (r).

A couple whose tendency is to rotate the body on which it acts, in a clockwise direction is known as a clockwise couple. Such a couple is also called +ve couple.

A couple whose tendency is to rotate the body on which it acts, in an anticlockwise direction is known as an anti-clockwise couple. Such a couple is also called -ve couple.



Moment of Couple:

The moment of a couple is the product of the force (ie, one of the forces of the two equal & opposite parallel forces) and the arm of the couple.

Mathematically,

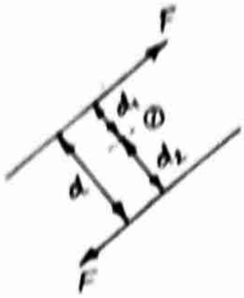
Moment of a couple = $F \times r$

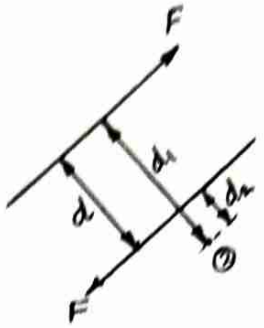
where F - magnitude of force

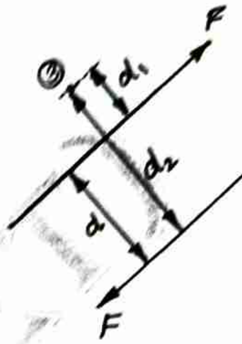
r - arm of the couple

Properties of a Couple {Ref: EM - Bhavikatti, D.S. Kumar

① If moment is taken about any point lying in the plane couple, then moment of the couple remains the same.





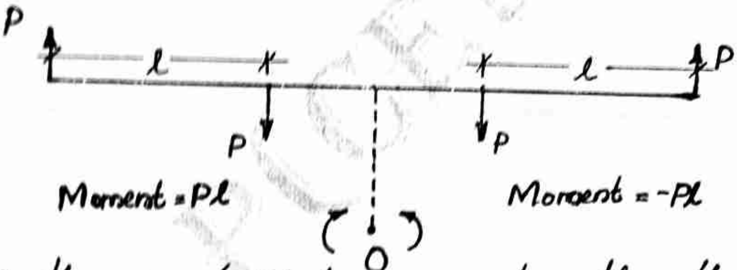


$M_1 = Fd_1 + Fd_2$
$= F(d_1 + d_2)$
$= Fd$

$M_2 = Fd_1 - Fd_2$
$= F(d_1 - d_2)$
$= F.d$

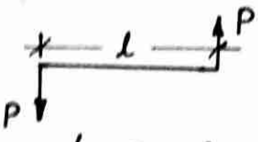
$M_3 = Fd_2 - Fd_1$
$= F(d_2 - d_1)$
$= F.d$

② Two coplanar couples, whose moments are equal and opposite balance each other.

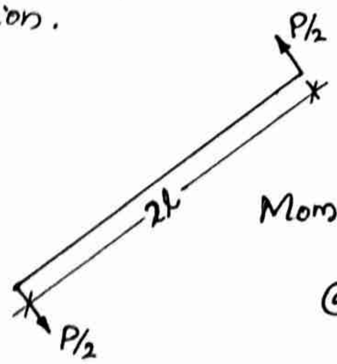


Since the couple balance each other, there will be no turning effect at point-'O'.

③ Any two couples will be equivalent if their moments are equ both in magnitude and direction.

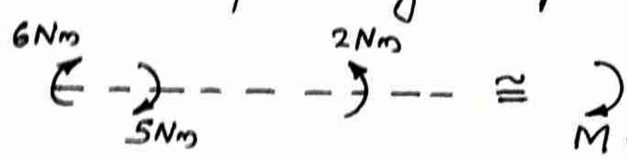


Moment = Pl (anticlockwise)



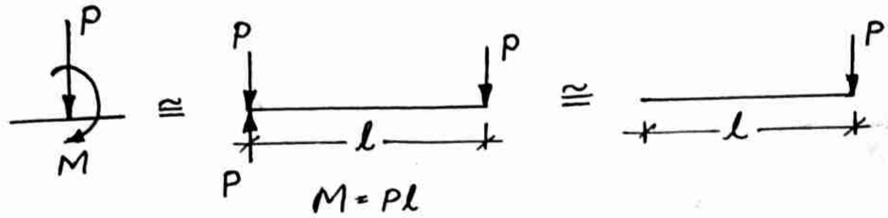
Moment $= P/2 \times 2l$
$= Pl$
(anticlockwise)

④ Algebraic sum of moments of a number of couples is equal to the moment of a single couple.

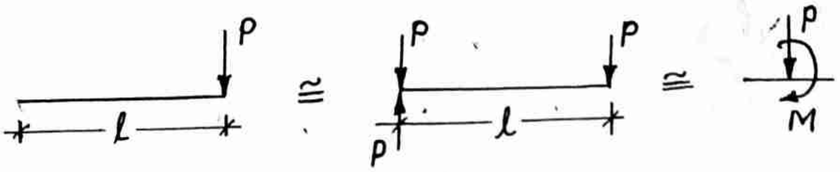


$M = 6 + 5 - 2 = 9 \text{ Nm}$

A single force -'P' and a couple-'M' acting in the same plane on a body cannot balance each other. However, they are together equivalent to a single force at a distance, $l = M/P$ from its original line of action.



Resolution of a force into a force & a couple

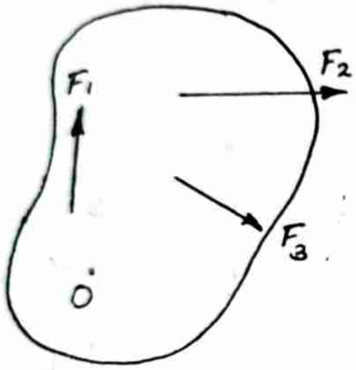


<u>Characteristics of a Couple</u>: {Ref: EM-D.S.Kumar

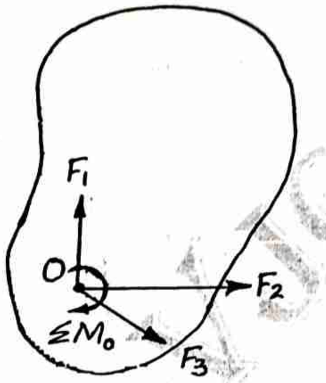
1) A couple consists of a pair of equal and opposite forces separated by a definite distance.
2) The algebraic sum of the forces constituting the couple is zero
3) The algebraic sum of the moment of the forces, constituting the couple, about any point is the same and equal to the moment of the couple itself.
4) A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
5) Any no: of co-planar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.
6) The translatory effect of a couple on a body is zero.
7) The effect of couple on a body remains unchanged if the couple:
   - rotated through an angle
   - shifted to any other position
   - replaced by another pair of forces whose rotational effect is same.

Resultant of Non-concurrent Force System: { Ref: EM- Bhavikatti

Resultant of a force system is the one which will have same rotational and translatory effect as the given system of forces. It may be a single force, a pure moment or a force and a moment.

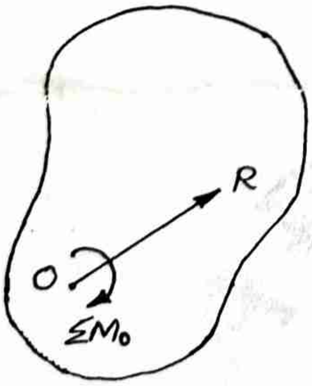


$\cong$

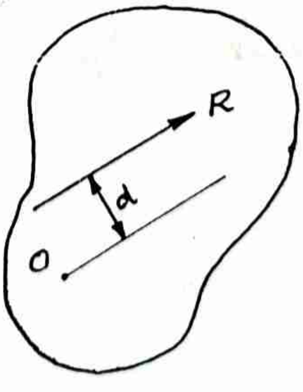


ΣM_o - Algebraic sum of the moments of the given forces about 'O'.

$\cong$



$\cong$



$\Sigma M_o = R \cdot d$